Precise Definition: We say \( \lim_{x \to a} f(x) = L \) if for every \( \varepsilon > 0 \) there is a \( \delta > 0 \) such that whenever \( 0 < |x - a| < \delta \) then \( |f(x) - L| < \varepsilon \).

“Working” Definition: We say \( \lim_{x \to a} f(x) = L \) if we can make \( f(x) \) as close to \( L \) as we want by taking \( x \) sufficiently close to \( a \) (on either side of \( a \)) without letting \( x = a \).

Right hand limit: \( \lim_{x \to a^+} f(x) = L \). This has the same definition as the limit except it requires \( x > a \).

Left hand limit: \( \lim_{x \to a^-} f(x) = L \). This has the same definition as the limit except it requires \( x < a \).

Relationship between the limit and one-sided limits
\[
\lim_{x \to a} f(x) = L \iff \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L \iff \lim_{x \to a} f(x) = L
\]

Properties
Assume \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) both exist and \( c \) is any number then,

1. \( \lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x) \)
2. \( \lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) \)
3. \( \lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x) \)
4. \( \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \) provided \( \lim_{x \to a} g(x) \neq 0 \)
5. \( \lim_{x \to a^} f(x)^n = [\lim_{x \to a^} f(x)]^n \)
6. \( \lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} \)

Basic Limit Evaluations at \( \pm \infty \)

Note: \( \text{sgn}(a) = 1 \) if \( a > 0 \) and \( \text{sgn}(a) = -1 \) if \( a < 0 \).

1. \( \lim_{x \to \infty} e^x = \infty \) & \( \lim_{x \to -\infty} e^x = 0 \)
2. \( \lim_{x \to \infty} \ln(x) = \infty \) & \( \lim_{x \to -\infty} \ln(x) = -\infty \)
3. \( \text{If } r > 0 \text{ then } \lim_{x \to \infty} \frac{b}{x^r} = 0 \)
4. \( \text{If } r > 0 \text{ and } x \) is real for negative \( x \)
   \[ \lim_{x \to -\infty} \frac{b}{x^r} = 0 \]
5. \( n \) even: \( \lim_{x \to \infty} x^n = \infty \)
6. \( n \) odd: \( \lim_{x \to \infty} x^n = \infty \) & \( \lim_{x \to -\infty} x^n = -\infty \)
7. \( n \) even: \( \lim_{x \to \infty} ax^n + \cdots + bx + c = \text{sgn}(a)\infty \)
8. \( n \) odd: \( \lim_{x \to \infty} ax^n + \cdots + bx + c = \text{sgn}(a)\infty \)
9. \( n \) odd: \( \lim_{x \to -\infty} ax^n + \cdots + cx + d = -\text{sgn}(a)\infty \)

Some Continuous Functions
Partial list of continuous functions and the values of \( x \) for which they are continuous.

1. Polynomials for all \( x \).
2. Rational function, except for \( x \)'s that give division by zero.
3. \( \sqrt[n]{x} \) (odd) for all \( x \).
4. \( \sqrt[n]{x} \) (even) for all \( x \geq 0 \).
5. \( e^x \) for all \( x \).
6. \( \ln(x) \) for \( x > 0 \).
7. \( \cos(x) \) and \( \sin(x) \) for all \( x \).
8. \( \tan(x) \) and \( \sec(x) \) provided \( x \neq \cdots, -\pi, -\frac{3\pi}{2}, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots \)
9. \( \cot(x) \) and \( \csc(x) \) provided \( x \neq \cdots, -2\pi, -\pi, 0, \pi, 2\pi, \cdots \)

Intermediate Value Theorem
Suppose that \( f(x) \) is continuous on \([a, b]\) and let \( M \) be any number between \( f(a) \) and \( f(b) \).
Then there exists a number \( c \) such that \( a < c < b \) and \( f(c) = M \).