Pari-GP reference card

(PARI-GP version 2.11.0)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: `gp`
To exit `gp`, type `quit`, `\q`, or `<C-D>` at prompt.

Help

describe function	`?`*function*
extended description	`??`*keyword*
list of relevant help topics	`???`*pattern*
name of GP-1.39 function f in GP-2.*	`whatnow`(f)

Input/Output

previous result, the result before	`%`, `%‘`, `%‘‘`, etc.
n-th result since startup	`%`n
separate multiple statements on line	`;`
extend statement on additional lines	`\`
extend statements on several lines	`{`seq_1`;` seq_2`;}`
comment	`/* ... */`
one-line comment, rest of line ignored	`\\ ...`

Metacommands & Defaults

set default d to *val*	`default`({d}, {*val*})
toggle timer on/off	`#`
print time for last result	`##`
print defaults	`\d`
set debug level to n	`\g` n
set memory debug level to n	`\gm` n
set n significant digits / bits	`\p` n, `\pb` n
set n terms in series	`\ps` n
quit GP	`\q`
print the list of PARI types	`\t`
print the list of user-defined functions	`\u`
read file into GP	`\r` *filename*

Debugger / break loop

get out of break loop	`break` or `<C-D>`
go up/down n frames	`dbg_up`({n}), `dbg_down`
set break point	`breakpoint()`
examine object o	`dbg_x`(o)
current error data	`dbg_err()`
number of objects on heap and their size	`getheap()`
total size of objects on PARI stack	`getstack()`

PARI Types & Input Formats

`t_INT`. Integers; hex, binary	±31; ±`0x1F`, ±`0b101`
`t_REAL`. Reals	±3.14, 6.022 E23
`t_INTMOD`. Integers modulo m	`Mod`(n,m)
`t_FRAC`. Rational Numbers	n/m
`t_FFELT`. Elt in finite field $\mathbf{F}_q$	`ffgen`(q,'t)
`t_COMPLEX`. Complex Numbers	$x+y*$`I`
`t_PADIC`. p-adic Numbers	$x+$`O`(p`^`k)
`t_QUAD`. Quadratic Numbers	$x+y*$`quadgen`(D,`'w`)
`t_POLMOD`. Polynomials modulo g	`Mod`(f,g)
`t_POL`. Polynomials	$a*x$`^`$n+\cdots+b$
`t_SER`. Power Series	$f+$`O`(x`^`k)
`t_RFRAC`. Rational Functions	f/g
`t_QFI`/`t_QFR`. Imag/Real binary quad. form	`Qfb`($a,b,c,\{d\}$)
`t_VEC`/`t_COL`. Row/Column Vectors	`[`x,y,z`]`, `[`x,y,z`]~`
`t_VEC` integer range	`[1..10]`
`t_VECSMALL`. Vector of small ints	`Vecsmall([`x,y,z`])`
`t_MAT`. Matrices	`[`$a,b;c,d$`]`
`t_LIST`. Lists	`List([`x,y,z`])`
`t_STR`. Strings	`"abc"`
`t_INFINITY`. $\pm\infty$	`+oo, -oo`

Reserved Variable Names

$\pi=3.14\ldots$, $\gamma=0.57\ldots$, $C=0.91\ldots$	`Pi`, `Euler`, `Catalan`
square root of -1	`I`
Landau's big-oh notation	`O`

Information about an Object

PARI type of object x	`type`(x)
length of x / size of x in memory	`#`x, `sizebyte`(x)
real precision / bit precision of x	`precision`(x), `bitprecision`
p-adic, series prec. of x	`padicprec`(x), `serprec`

Operators

basic operations	`+`, `-` , `*`, `/`, `^`, `sqr`
`i=i+1`, `i=i-1`, `i=i*j`, ...	`i++`, `i--`, `i*=j`,...
euclidean quotient, remainder	x`\/`y, x`\`y, x`%`y, `divrem`(x,y)
shift x left or right n bits	x`<<`n, x`>>`n or `shift`($x,\pm n$)
multiply by 2^n	`shiftmul`(x,n)
comparison operators	`<=`, `<`, `>=`, `>`, `==`, `!=`, `===`, `lex`, `cmp`
boolean operators (or, and, not)	`||`, `&&`, `!`
bit operations	`bitand`, `bitneg`, `bitor`, `bitxor`, `bitnegimply`
maximum/minimum of x and y	`max`, `min`(x,y)
sign of $x=-1,0,1$	`sign`(x)
binary exponent of x	`exponent`(x)
derivative of f	f`'`
differential operator	`diffop`($f,v,d,\{n=1\}$)
quote operator (formal variable)	`'x`
assignment	`x =` *value*
simultaneous assignment $x\leftarrow v_1$, $y\leftarrow v_2$	`[x,y] = v`

Select Components

n-th component of x	`component`(x,n)
n-th component of vector/list x	x`[`n`]`
components $a,a+1,\ldots,b$ of vector x	x`[`a`..`b`]`
(m,n)-th component of matrix x	x`[`m,n`]`
row m or column n of matrix x	x`[`m`,]`, x`[,`n`]`
numerator/denominator of x	`numerator`(x), `denominator`

Random Numbers

random integer/prime in $[0,N[$	`random`(N), `randomprime`
get/set random seed	`getrand`, `setrand`(s)

Conversions

to vector, matrix, vec. of small ints	`Col`/`Vec`,`Mat`,`Vecsmall`
to list, set, map, string	`List`, `Set`, `Map`, `Str`
create PARI object (x mod y)	`Mod`(x,y)
make x a polynomial of v	`Pol`($x,\{v\}$)
as `Pol`, etc., starting with constant term	`Polrev`, `Vecrev`, `Colrev`
make x a power series of v	`Ser`($x,\{v\}$)
string from bytes / from format+args	`Strchr`, `Strprintf`
TeX string	`Strtex`(x)
convert x to simplest possible type	`simplify`(x)
object x with real precision n	`precision`(x,n)
object x with bit precision n	`bitprecision`(x,n)
set precision to p digits in dynamic scope	`localprec`(p)
set precision to p bits in dynamic scope	`localbitprec`(p)

Conjugates and Lifts

conjugate of a number x	`conj`(x)
norm of x, product with conjugate	`norm`(x)
L^p norm of x (L^∞ if no p)	`normlp`($x,\{p\}$)
square of L^2 norm of x	`norml2`(x)
lift of x from Mods and p-adics	`lift`, `centerlift`(x)
recursive lift	`liftall`
lift all `t_INT` and `t_PADIC` ($\to$`t_INT`)	`liftint`
lift all `t_POLMOD` ($\to$`t_POL`)	`liftpol`

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. `cmp`)

intersection of sets x and y	`setintersect`(x,y)
set of elements in x not belonging to y	`setminus`(x,y)
union of sets x and y	`setunion`(x,y)
does y belong to the set x	`setsearch`($x,y,\{flag\}$)
set of all $f(x,y)$, $x\in X$, $y\in Y$	`setbinop`(f,X,Y)
is x a set ?	`setisset`(x)

Lists. create empty list: $L=$ `List()`

append x to list L	`listput`($L,x,\{i\}$)
remove i-th component from list L	`listpop`($L,\{i\}$)
insert x in list L at position i	`listinsert`(L,x,i)
sort the list L in place	`listsort`($L,\{flag\}$)

Maps. create empty dictionnary: $M=$ `Map()`

attach value v to key k	`mapput`(M,k,v)
recover value attach to key k or error	`mapget`(M,k)
is key k in the dict ? (set v to $M(k)$)	`mapisdefined`($M,k,\{\&v\}$)
remove k from map domain	`mapdelete`(M,k)

GP Programming

User functions and closures

x,y are formal parameters; y defaults to `Pi` if parameter opitted;
z,t are local variables (lexical scope), z initialized to 1.

`fun(x, y=Pi) = my(z=1, t);` *seq*

`fun = (x, y=Pi) -> my(z=1, t);` *seq*

attach a help message to f	`addhelp`(f)
undefine symbol s (also kills help)	`kill`(s)

Control Statements (X: formal parameter in expression *seq*)

if $a\neq 0$, evaluate seq_1, else seq_2	`if`($a,\{seq_1\},\{seq_2\}$)
eval. *seq* for $a\le X\le b$	`for`($X=a,b,seq$)
...for primes $a\le X\le b$	`forprime`($X=a,b,seq$)
...for primes $\equiv a \pmod q$	`forprimestep`($X=a,b,q,seq$)
...for composites $a\le X\le b$	`forcomposite`($X=a,b,seq$)
...for $a\le X\le b$ stepping s	`forstep`($X=a,b,s,seq$)
...for X dividing n	`fordiv`(n,X,seq)
...$X=[n,factor(n)]$, $a\le n\le b$	`forfactored`($X=a,b,seq$)
...as above, n squarefree	`forsquarefree`($X=a,b,seq$)
...$X=[d,factor(d)]$, $d\mid n$	`fordivfactored`(n,X,seq)
multivariable `for`, lex ordering	`forvec`($X=v,seq$)
loop over partitions of n	`forpart`($p=n,seq$)
...permutations of S	`forperm`(S,p,seq)
...subsets of $\{1,\ldots,n\}$	`forsubset`(n,p,seq)
...k-subsets of $\{1,\ldots,n\}$	`forsubset`($[n,k],p,seq$)
...vectors v, $q(v)\le B$; $q>0$	`forqfvec`(v,q,b,seq)
...$H<G$ finite abelian group	`forsubgroup`($H=G$)
evaluate *seq* until $a\neq 0$	`until`(a,seq)
while $a\neq 0$, evaluate *seq*	`while`(a,seq)
exit n innermost enclosing loops	`break`($\{n\}$)
start new iteration of n-th enclosing loop	`next`($\{n\}$)
return x from current subroutine	`return`($\{x\}$)

Exceptions, warnings

raise an exception / warn	`error()`, `warning()`
type of error message E	`errname`(E)
try seq_1, evaluate seq_2 on error	`iferr`(seq_1, E, seq_2)

Functions with closure arguments / results

select from v according to f	`select`(f, v)
apply f to all entries in v	`apply`(f, v)
evaluate $f(a_1, \ldots, a_n)$	`call`(f, a)
evaluate $f(\ldots f(f(a_1, a_2), a_3) \ldots, a_n)$	`fold`(f, a)
calling function as closure	`self()`

Sums & Products

sum $X = a$ to $X = b$, initialized at x	`sum`$(X = a, b, expr, \{x\})$
sum entries of vector v	`vecsum`(v)
product of all vector entries	`vecprod`(v)
sum *expr* over divisors of n	`sumdiv`$(n, X, expr)$
... assuming *expr* multiplicative	`sumdivmult`$(n, X, expr)$
product $a \le X \le b$, initialized at x	`prod`$(X = a, b, expr, \{x\})$
product over primes $a \le X \le b$	`prodeuler`$(X = a, b, expr)$

Sorting

sort x by k-th component	`vecsort`$(x, \{k\}, \{fl = 0\})$
min. m of x ($m = x[i]$), max.	`vecmin`$(x, \{\&i\})$, `vecmax`
does y belong to x, sorted wrt. f	`vecsearch`$(x, y, \{f\})$

Input/Output

print with/without `\n`, TeX format	`print`, `print1`, `printtex`
pretty print matrix	`printp`
print fields with separator	`printsep`$(sep, \ldots)$,, `printsep1`
formatted printing	`printf()`
write *args* to file	`write`, `write1`, `writetex`$(file, args)$
write x in binary format	`writebin`$(file, x)$
read file into GP	`read`$(\{file\})$
... return as vector of lines	`readvec`$(\{file\})$
... return as vector of strings	`readstr`$(\{file\})$
read a string from keyboard	`input()`

Files and file descriptors

File descriptors allows efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in `r`(ead), `w`(rite) or `a`(ppend) mode.

get descriptor n for file *path* in given *mode*	`fileopen`$(path, mode)$
... from shell *cmd* output (pipe)	`fileextern`(cmd)
close descriptor	`fileclose`(n)
commit pending write operations	`fileflush`(n)
read logical line from file	`fileread`(n)
... raw line from file	`filereadstr`(n)
write s`\n` to file	`filewrite`(n, s)
... write s to file	`filewrite1`(n, s)

Timers

CPU time in *ms* and reset timer	`gettime()`
CPU time in *ms* since gp startup	`getabstime()`
time in *ms* since UNIX Epoch	`getwalltime()`
timeout command after s seconds	`alarm`$(s, expr)$

Interface with system

allocates a new stack of s bytes	`allocatemem`$(\{s\})$
alias *old* to *new*	`alias`(new, old)
install function from library	`install`$(f, code, \{gpf\}, \{lib\})$
execute system command a	`system`(a)
... and feed result to GP	`extern`(a)
... returning GP string	`externstr`(a)

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get `$VAR` from environment	`getenv("VAR")`
expand env. variable in string	`Strexpand`(x)

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \ldots, x[n]$	`parapply`(f, x)
evaluate closures $f[1], \ldots, f[n]$	`pareval`(f)
as `select`	`parselect`$(f, A, \{flag\})$
as `sum`	`parsum`$(i = a, b, expr, \{x\})$
as `vector`	`parvector`$(n, i, \{expr\})$
eval f for $i = a, \ldots, b$	`parfor`$(i = a, \{b\}, f, \{r\}, \{f_2\})$
... for p prime in $[a, b]$	`parforprime`$(p = a, \{b\}, f, \{r\}, \{f_2\})$
... multivariate	`parforvec`$(X = v, f, \{r\}, \{f_2\}, \{flag\})$
declare x as inline (allows to use as global)	`inline`(x)
stop inlining	`uninline()`

Linear Algebra

dimensions of matrix x	`matsize`(x)
multiply two matrices	x `*` y
... assuming result is diagonal	`matmultodiagonal`(x, y)
concatenation of x and y	`concat`$(x, \{y\})$
extract components of x	`vecextract`$(x, y, \{z\})$
transpose of vector or matrix x	`mattranspose`(x) or x`~`
adjoint of the matrix x	`matadjoint`(x)
eigenvectors/values of matrix x	`mateigen`(x)
characteristic/minimal polynomial of x	`charpoly`(x), `minpoly`
trace/determinant of matrix x	`trace`(x), `matdet`
permanent of matrix x	`matpermanent`(x)
Frobenius form of x	`matfrobenius`(x)
QR decomposition	`matqr`(x)
apply `matqr`'s transform to v	`mathouseholder`(Q, v)

Constructors & Special Matrices

$\{g(x)\colon x \in v \text{ s.t. } f(x)\}$	`[g(x) | x <- v, f(x)]`
$\{x\colon x \in v \text{ s.t. } f(x)\}$	`[x | x <- v, f(x)]`
$\{g(x)\colon x \in v\}$	`[g(x) | x <- v]`
row vec. of *expr* eval'ed at $1 \le i \le n$	`vector`$(n, \{i\}, \{expr\})$
col. vec. of *expr* eval'ed at $1 \le i \le n$	`vectorv`$(n, \{i\}, \{expr\})$
vector of small ints	`vectorsmall`$(n, \{i\}, \{expr\})$
$[c, c \cdot x, \ldots, c \cdot x^n]$	`powers`$(x, n, \{c = 1\})$
matrix $1 \le i \le m$, $1 \le j \le n$	`matrix`$(m, n, \{i\}, \{j\}, \{expr\})$
define matrix by blocks	`matconcat`(B)
diagonal matrix with diagonal x	`matdiagonal`(x)
is x diagonal?	`matisdiagonal`(x)
$x \cdot$ `matdiagonal`(d)	`matmuldiagonal`(x, d)
$n \times n$ identity matrix	`matid`(n)
Hessenberg form of square matrix x	`mathess`(x)
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	`mathilbert`(n)
$n \times n$ Pascal triangle	`matpascal`$(n - 1)$
companion matrix to polynomial x	`matcompanion`(x)
Sylvester matrix of x	`polsylvestermatrix`(x)

Gaussian elimination

kernel of matrix x	`matker`$(x, \{flag\})$
intersection of column spaces of x and y	`matintersect`(x, y)
solve $MX = B$ (M invertible)	`matsolve`(M, B)
one sol of $M * X = B$	`matinverseimage`(M, B)
basis for image of matrix x	`matimage`(x)
columns of x *not* in `matimage`	`matimagecompl`(x)
supplement columns of x to get basis	`matsupplement`(x)
rows, cols to extract invertible matrix	`matindexrank`(x)
rank of the matrix x	`matrank`(x)
solve $MX = B$ mod D	`matsolvemod`(M, D, B)
image mod D	`matimagemod`(M, D)
kernel mod D	`matkermod`(M, D)
inverse mod D	`matinvmod`(M, D)
determinant mod D	`matdetmod`(M, D)

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$	`qfeval`$(\{Q = id\}, x, y)$
evaluate ${}^t x Q x$	`qfeval`$(\{Q = id\}, x)$
signature of quad form ${}^t y * x * y$	`qfsign`(x)
decomp into squares of ${}^t y * x * y$	`qfgaussred`(x)
eigenvalues/vectors for real symmetric x	`qfjacobi`(x)

HNF and SNF

upper triangular Hermite Normal Form	`mathnf`(x)
HNF of x where d is a multiple of $\det(x)$	`mathnfmod`(x, d)
multiple of $\det(x)$	`matdetint`(x)
HNF of (x | `diagonal`(D))	`mathnfmodid`(x, D)
elementary divisors of x	`matsnf`(x)
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	`poldiscreduced`(f)
integer kernel of x	`matkerint`(x)
$\mathbf{Z}$-module $\leftrightarrow$ $\mathbf{Q}$-vector space	`matrixqz`(x, p)

Lattices

LLL-algorithm applied to columns of x	`qflll`$(x, \{flag\})$
... for Gram matrix of lattice	`qflllgram`$(x, \{flag\})$
find up to m sols of `qfnorm`$(x, y) \le b$	`qfminim`(x, b, m)
v, $v[i]$:=number of y s.t. `qfnorm`$(x, y) = i$	`qfrep`$(x, B, \{flag\})$
perfection rank of x	`qfperfection`(x)
find isomorphism between q and Q	`qfisom`(q, Q)
precompute for isomorphism test with q	`qfisominit`(q)
automorphism group of q	`qfauto`(q)
convert `qfauto` for GAP/Magma	`qfautoexport`$(G, \{flag\})$
orbits of V under $G \subset \mathrm{GL}(V)$	`qforbits`(G, V)

Polynomials & Rational Functions

all defined polynomial variables	`variables()`
get var. of highest priority (higher than v)	`varhigher`$(name, \{v\})$
... of lowest priority (lower than v)	`varlower`$(name, \{v\})$

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Coefficients, variables and basic operators

degree of f	poldegree(f)
coef. of degree n of f, leading coef.	polcoef(f, n), pollead
main variable / all variables in f	variable(f), variables(f)
replace x by y in f	subst(f, x, y)
evaluate f replacing vars by their value	eval(f)
replace polynomial expr. $T(x)$ by y in f	substpol(f, T, y)
replace $x_1, \ldots, x_n$ by $y_1, \ldots, y_n$ in f	substvec(f, x, y)
reciprocal polynomial $x^{\deg f} f(1/x)$	polrecip(f)
gcd of coefficients of f	content(f)
derivative of f w.r.t. x	deriv($f, \{x\}$)
formal integral of f w.r.t. x	intformal($f, \{x\}$)
formal sum of f w.r.t. x	sumformal($f, \{x\}$)

Constructors & Special Polynomials

interpolating pol. eval. at a	polinterpolate($X, \{Y\}, \{a\}$)
P_n, T_n/U_n, H_n	pollegendre, polchebyshev, polhermite
n-th cyclotomic polynomial Φ_n	polcyclo($n, \{v\}$)
return n if $f = \Phi_n$, else 0	poliscyclo(f)
is f a product of cyclotomic polynomials?	poliscycloprod(f)
Zagier's polynomial of index (n, m)	polzagier(n, m)

Resultant, elimination

discriminant of polynomial f	poldisc(f)
find factors of poldisc(f)	poldiscfactors(f)
resultant $R = \mathrm{Res}_v(f, g)$	polresultant($f, g, \{v\}$)
$[u, v, R]$, $xu + yv = \mathrm{Res}_v(f, g)$	polresultantext($x, y, \{v\}$)
solve Thue equation $f(x, y) = a$	thue($t, a, \{sol\}$)
initialize t for Thue equation solver	thueinit(f)

Roots and Factorization (Complex/Real)

complex roots of f	polroots(f)
bound complex roots of f	polrootsbound(f)
number of real roots of f (in $[a, b]$)	polsturm($f, \{[a, b]\}$)
real roots of f (in $[a, b]$)	polrootsreal($f, \{[a, b]\}$)
complex embeddings of t_POLMOD z	conjvec(z)

Roots and Factorization (Finite fields)

factor f mod p, roots	factormod(f, p), polrootsmod
factor f over $\mathbf{F}_p[x]/(T)$, roots	factormod($f, [T, p]$), polrootsmod
squarefree factorization of f in $\mathbf{F}_q[x]$	factormodSQF($f, \{D\}$)
distinct degree factorization of f in $\mathbf{F}_q[x]$	factormodDDF($f, \{D\}$)

Roots and Factorization (p-adic fields)

factor f over $\mathbf{Q}_p$, roots	factorpadic(f, p, r), polrootspadic
p-adic root of f congruent to a mod p	padicappr(f, a)
Newton polygon of f for prime p	newtonpoly(f, p)
Hensel lift $A/\mathrm{lc}(A) = \prod_i B[i]$ mod p^e	polhensellift(A, B, p, e)
extensions of $\mathbf{Q}_p$ of degree N	padicfields(p, N)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n	polsym(f, n)
Graeffe transform of f, $g(x^2) = f(x)f(-x)$	polgraeffe(f)
factor f over coefficient field	factor(f)
cyclotomic factors of $f \in \mathbf{Q}[X]$	polcyclofactors(f)

Finite Fields

A finite field is encoded by any element (t_FFELT).

find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	ffinit($p, n, \{x\}$)
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$	t = ffgen(T, 't)
... indirectly, with implicit T	t = ffgen(q, 't); T = t.mod
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$	m = ffembed(a, b)
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$,	ffextend(a, P)
evaluate map m on x	ffmap(m, x)
inverse map of m	ffinvmap(m)
compose maps $m \circ n$	ffcompomap(m, n)
F^n over $\mathbf{F}_q \ni a$	fffrobenius(a, n)
$\#\{$monic irred. $T \in \mathbf{F}_q[x], \deg T = n\}$	ffnbirred(q, n)

Formal & p-adic Series

truncate power series or p-adic number	truncate(x)
valuation of x at p	valuation(x, p)

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	taylor(f, x)
Laurent series expansion around 0 up to x^k	laurentseries(f, k)
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	serconvol(a, b)
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	serlaplace(f)
reverse power series F so $F(f(x)) = x$	serreverse(f)
remove terms of degree $< n$ in f	serchop(f, n)
Dirichlet series multiplication / division	dirmul, dirdiv(x, y)
Dirichlet Euler product (b terms)	direuler($p = a, b, expr$)

Transcendental and p-adic Functions

real, imaginary part of x	real(x), imag(x)
absolute value, argument of x	abs(x), arg(x)
square/nth root of x	sqrt(x), sqrtn($x, n, \{$&$z\}$)
trig functions	sin, cos, tan, cotan, sinc
inverse trig functions	asin, acos, atan
hyperbolic functions	sinh, cosh, tanh, cotanh
inverse hyperbolic functions	asinh, acosh, atanh
$\log(x)$, $\log(1 + x)$, e^x, $e^x - 1$	log, log1p, exp, expm1
Euler Γ function, $\log \Gamma$, Γ'/Γ	gamma, lngamma, psi
half-integer gamma function $\Gamma(n + 1/2)$	gammah(n)
Riemann's zeta $\zeta(s) = \sum n^{-s}$	zeta(s)
Hurwitz's $\zeta(s, x) = \sum (n + x)^{-s}$	zetahurwitz(s, x)
multiple zeta value (MZV), $\zeta(s_1, \ldots, s_k)$	zetamult($s, \{T\}$)
... init T for MZV with $s_1 + \ldots + s_k \leq w$	zetamultinit(w)
all MZVs for all weights $\sum s_i \leq n$	zetamultall(n)
convert MZV id to $[s_1, \ldots, s_k]$	zetamultconvert($f, \{flag\}$)
incomplete Γ function ($y = \Gamma(s)$)	incgam($s, x, \{y\}$)
complementary incomplete Γ	incgamc(s, x)
$\int_x^\infty e^{-t}\,dt/t$, $\quad (2/\sqrt{\pi}) \int_x^\infty e^{-t^2}\,dt$	eint1, erfc
dilogarithm of x	dilog(x)
m-th polylogarithm of x	polylog($m, x, \{flag\}$)
U-confluent hypergeometric function	hyperu(a, b, u)
Bessel $J_n(x)$, $J_{n+1/2}(x)$	besselj(n, x), besseljh(n, x)
Bessel I_ν, K_ν, H^1_ν, H^2_ν, N_ν	(bessel)i, k, h1, h2, n
Lambert W: x s.t. $xe^x = y$	lambertw(y)
Teichmuller character of p-adic x	teichmuller(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm$oo (decreasing at least as x^{-2}) or

$(x - a)^{-\alpha}$ singularity	$[a, \alpha]$
exponential decrease $e^{-\alpha\lvert x\rvert}$	$[\pm\infty, \alpha]$, $\alpha > 0$
slow decrease $\lvert x\rvert^\alpha$	$\ldots \alpha < -1$
oscillating as $\cos(kx)$)	$\alpha = k$I, $k > 0$
oscillating as $\sin(kx)$)	$\alpha = -k$I, $k > 0$
numerical integration	intnum($x = a, b, f, \{T\}$)
weights T for intnum	intnuminit($a, b, \{m\}$)
weights T incl. kernel K	intfuncinit($a, b, K, \{m\}$)
integrate $(2i\pi)^{-1} f$ on circle $\lvert z - a\rvert = R$	intcirc($x = a, R, f, \{T\}$)

Other integration methods

n-point Gauss-Legendre	intnumgauss($x = a, b, f, \{n\}$)
weights for n-point Gauss-Legendre	intnumgaussinit($\{n\}$)
Romberg integration (low accuracy)	intnumromb($x = a, b, f, \{flag\}$)

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy)	suminf($n = a, expr$)
sum of alternating/positive series	sumalt, sumpos
sum of series using Euler-Maclaurin	sumnum($n = a, f, \{T\}$)
$\sum_{n \geq a} F(n)$, F rational function	sumnumrat(F, a)
$\ldots \sum_{n \geq a} (-1)^n F(n)$	sumaltrat(F, a)
$\ldots \sum_{p \geq a} F(p^s)$	sumeulerrat($F, \{s = 1\}, \{a = 2\}$)
weights for sumnum, a as in DE	sumnuminit($\{\infty, a\}$)
sum of series by Monien summation	sumnummonien($n = a, f, \{T\}$)
weights for sumnummonien	sumnummonieninit($\{\infty, a\}$)
sum of series using Abel-Plana	sumnumap($n = a, f, \{T\}$)
weights for sumnumap, a as in DE	sumnumapinit($\{\infty, a\}$)
sum of series using Lagrange	sumnumlagrange($n = a, f, \{T\}$)
weights for sumnumlagrange	sumnumlagrangeinit

Products

product $a \leq X \leq b$, initialized at x	prod($X = a, b, expr, \{x\}$)
product over primes $a \leq X \leq b$	prodeuler($X = a, b, expr$)
infinite product $a \leq X \leq \infty$	prodinf($X = a, expr$)
$\prod_{n \geq a} F(n)$, F rational function	prodnumrat(F, a)
$\ldots \prod_{p \geq a} F(p^s)$	prodeulerrat($F, \{s = 1\}, \{a = 2\}$)

Other numerical methods

real root of f in $[a, b]$; bracketed root	solve($X = a, b, f$)
... by interval splitting	solvestep($X = a, b, f, \{flag = 0\}$)
limit of $f(t)$, $t \to \infty$	limitnum(f, {k}, {alpha})
asymptotic expansion of f at ∞	asympnum(f, {k}, {alpha})
numerical derivation w.r.t x: $f'(a)$	derivnum($x = a, f$)
evaluate continued fraction F at t	contfraceval($F, t, \{L\}$)
power series to cont. fraction (L terms)	contfracinit($S, \{L\}$)
Padé approximant (deg. denom. $\leq B$)	bestapprPade($S, \{B\}$)

Elementary Arithmetic Functions

vector of binary digits of $\lvert x\rvert$	`binary`(x)
bit number n of integer x	`bittest`(x,n)
Hamming weight of integer x	`hammingweight`(x)
digits of integer x in base B	`digits`$(x,\{B=10\})$
sum of digits of integer x in base B	`sumdigits`$(x,\{B=10\})$
integer from digits	`fromdigits`$(v,\{B=10\})$
ceiling/floor/fractional part	`ceil`, `floor`, `frac`
round x to nearest integer	`round`$(x,\{\&e\})$
truncate x	`truncate`$(x,\{\&e\})$
gcd/LCM of x and y	`gcd`(x,y), `lcm`(x,y)
gcd of entries of a vector/matrix	`content`(x)

Primes and Factorization

extra prime table	`addprimes()`
add primes in v to prime table	`addprimes`(v)
remove primes from prime table	`removeprimes`(v)
Chebyshev $\pi(x)$, n-th prime p_n	`primepi`(x), `prime`(n)
vector of first n primes	`primes`(n)
smallest prime $\geq x$	`nextprime`(x)
largest prime $\leq x$	`precprime`(x)
factorization of x	`factor`$(x,\{lim\})$
...selecting specific algorithms	`factorint`$(x,\{flag=0\})$
$n=df^2$, d squarefree/fundamental	`core`$(n,\{fl\})$, `coredisc`
certificate for (prime) N	`primecert`(N)
verifies a certificate c	`primecertisvalid`(c)
convert certificate to Magma/PRIMO	`primecertexport`
recover x from its factorization	`factorback`$(f,\{e\})$
$x\in\mathbf{Z}$, $\lvert x\rvert\leq X$, $\gcd(N,P(x))\geq N$	`zncoppersmith`$(P,N,X,\{B\})$
divisors of N in residue class r mod s	`divisorslenstra`(N,r,s)

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	`omega`(n), `bigomega`
divisors of n / number of divisors $\tau(n)$	`divisors`(n), `numdiv`
sum of (k-th powers of) divisors of n	`sigma`$(n,\{k\})$
Möbius μ-function	`moebius`(x)
Ramanujan's τ-function	`ramanujantau`(x)

Combinatorics

factorial of x	`x!` or `factorial`(x)
binomial coefficient $\binom{x}{k}$	`binomial`$(x,\{k\})$
Bernoulli number B_n as real/rational	`bernreal`(n), `bernfrac`
Bernoulli polynomial $B_n(x)$	`bernpol`$(n,\{x\})$
n-th Fibonacci number	`fibonacci`(n)
Stirling numbers $s(n,k)$ and $S(n,k)$	`stirling`$(n,k,\{flag\})$
number of partitions of n	`numbpart`(n)
k-th permutation on n letters	`numtoperm`(n,k)
convert permutation to (n,k) form	`permtonum`(v)
order of permutation p	`permorder`(p)
signature of permutation p	`permsign`(p)

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	`eulerphi`(x)
multiplicative order of x (divides o)	`znorder`$(x,\{o\})$, `fforder`
primitive root mod q / x`.mod`	`znprimroot`(q), `ffprimroot`(x)
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	`znstar`(n)
discrete logarithm of x in base g	`znlog`$(x,g,\{o\})$, `fflog`
Kronecker-Legendre symbol $(\frac{x}{y})$	`kronecker`(x,y)
quadratic Hilbert symbol (at p)	`hilbert`$(x,y,\{p\})$

Miscellaneous

integer square / n-th root of x	`sqrtint`(x), `sqrtnint`(x,n)
largest integer e s.t. $b^e\leq b$, $e=\lfloor\log_b(x)\rfloor$	`logint`$(x,b,\{\&z\})$
CRT: solve $z\equiv x$ and $z\equiv y$	`chinese`(x,y)
minimal u,v so $xu+yv=\gcd(x,y)$	`gcdext`(x,y)
continued fraction of x	`contfrac`$(x,\{b\},\{lmax\})$
last convergent of continued fraction x	`contfracpnqn`(x)
rational approximation to x (den. $\leq B$)	`bestappr`$(x,\{B\}k)$
recognize $x\in\mathbf{C}$ as polmod mod $T\in\mathbf{Z}[X]$	`bestapprnf`(x,T)

Characters

Let $cyc=[d_1,\dots,d_k]$ represent an abelian group $G=\oplus(\mathbf{Z}/d_j\mathbf{Z})\cdot g_j$ or any structure G affording a `.cyc` method; e.g. `znstar`$(q,1)$ for Dirichlet characters. A character χ is coded by $[c_1,\dots,c_k]$ such that $\chi(g_j)=e(n_j/d_j)$.

$\chi\cdot\psi$; χ^{-1}; $\chi\cdot\psi^{-1}$; χ^k	`charmul`, `charconj`, `chardiv`,, `charpow`
order of χ	`charorder`(cyc,χ)
kernel of χ	`charker`(cyc,χ)
$\chi(x)$, G a GP group structure	`chareval`$(G,\chi,x,\{z\})$
Galois orbits of characters	`chargalois`(G)

Dirichlet Characters

initialize $G=(\mathbf{Z}/q\mathbf{Z})^*$	`G = znstar`$(q,1)$
convert datum D to $[G,\chi]$	`znchar`(D)
is χ odd?	`zncharisodd`(G,χ)
real $\chi\to$ Kronecker symbol $(D/.)$	`znchartokronecker`(G,χ)
conductor of χ	`zncharconductor`(G,χ)
$[G_0,\chi_0]$ primitive attached to χ	`znchartoprimitive`(G,χ)
induce $\chi\in\hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	`zncharinduce`(G,χ,N)
χ_p	`znchardecompose`(G,χ,p)
$\prod_{p\mid(Q,N)}\chi_p$	`znchardecompose`(G,χ,Q)
complex Gauss sum $G_a(\chi)$	`znchargauss`(G,χ)

Conrey labelling

Conrey label $m\in(\mathbf{Z}/q\mathbf{Z})^*\to$ character	`znconreychar`(G,m)
character $\to$ Conrey label	`znconreyexp`(G,χ)
log on Conrey generators	`znconreylog`(G,m)
conductor of χ (χ_0 primitive)	`znconreyconductor`$(G,\chi,\{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field?	`isfundamental`(x)
is x a prime?	`isprime`(x)
is x a strong pseudo-prime?	`ispseudoprime`(x)
is x square-free?	`issquarefree`(x)
is x a square?	`issquare`$(x,\{\&n\})$
is x a perfect power?	`ispower`$(x,\{k\},\{\&n\})$
is x a perfect power of a prime? ($x=p^n$)	`isprimepower`$(x,\&n\})$
...of a pseudoprime?	`ispseudoprimepower`$(x,\&n\})$
is x powerful?	`ispowerful`(x)
is x a totient? ($x=\varphi(n)$)	`istotient`$(x,\{\&n\})$
is x a polygonal number? ($x=P(s,n)$)	`ispolygonal`$(x,s,\{\&n\})$
is pol irreducible?	`polisirreducible`(pol)

Graphic Functions

crude graph of $expr$ between a and b	`plot`$(X=a,b,expr)$

High-resolution plot (immediate plot)

plot $expr$ between a and b	`ploth`$(X=a,b,expr,\{flag\},\{n\})$
plot points given by lists lx, ly	`plothraw`$(lx,ly,\{flag\})$
terminal dimensions	`plothsizes()`

Rectwindow functions

init window w, with size x,y	`plotinit`(w,x,y)
erase window w	`plotkill`(w)
copy w to w_2 with offset (dx,dy)	`plotcopy`(w,w_2,dx,dy)
clips contents of w	`plotclip`(w)
scale coordinates in w	`plotscale`(w,x_1,x_2,y_1,y_2)
`ploth` in w	`plotrecth`$(w,X=a,b,expr,\{flag\},\{n\})$
`plothraw` in w	`plotrecthraw`$(w,data,\{flag\})$
draw window w_1 at (x_1,y_1), ...	`plotdraw`$([[w_1,x_1,y_1],\ldots])$

Low-level Rectwindow Functions

set current drawing color in w to c	`plotcolor`(w,c)
current position of cursor in w	`plotcursor`(w)
write s at cursor's position	`plotstring`(w,s)
move cursor to (x,y)	`plotmove`(w,x,y)
move cursor to $(x+dx,y+dy)$	`plotrmove`(w,dx,dy)
draw a box to (x_2,y_2)	`plotbox`(w,x_2,y_2)
draw a box to $(x+dx,y+dy)$	`plotrbox`(w,dx,dy)
draw polygon	`plotlines`$(w,lx,ly,\{flag\})$
draw points	`plotpoints`(w,lx,ly)
draw line to $(x+dx,y+dy)$	`plotrline`(w,dx,dy)
draw point $(x+dx,y+dy)$	`plotrpoint`(w,dx,dy)
draw point $(x+dx,y+dy)$	`plotrpoint`(w,dx,dy)

Convert to Postscript or Scalable Vector Graphics

The format f is either `"ps"` or `"svg"`.

as `ploth`	`plothexport`$(f,X=a,b,expr,\{flag\},\{n\})$
as `plothraw`	`plothrawexport`$(f,lx,ly,\{flag\})$
as `plotdraw`	`plotexport`$(f,[[w_1,x_1,y_1],\ldots])$

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